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LETTER TO THE EDITOR

Dynamic universality classes for diffusion-limited aggregation and lattice animals

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Abstract. A recent application of the real space renormalisation group to the problem of anomalous diffusion on Witten-Sander clusters is generalised and we demonstrate that these clusters lie in a distinct dynamic universality class from random lattice animals.

A feature arising from the renormalisation group (RG) theory (Wilson and Kogut 1974) is that diverse systems undergoing continuous phase transitions, characterised by a diverging controlling length, partition themselves into universality classes. Within a universality class, systems which seem, in a superficial sense at least, very different to each other have critical behaviour which is essentially the same, the apparent differences becoming irrelevant as one approaches criticality.

The critical behaviour for a particular system is governed by a set of critical exponents and a subset of these are the same for all the constituent systems of a particular universality class. It is convenient to divide the set of universality classes into two groups: static and dynamic†. Systems within the same static universality class will have the same exponents characterising the singularities in the static properties and equivalently for dynamic universality classes. We recall that two systems may be in the same static universality class but in distinct dynamic ones.

In RG theory one may represent the set of universality classes by an isomorphic set of critical surfaces embedded in a parameter space. Systems with the same critical behaviour will flow towards the same critical point.

In this letter we generalise the real space renormalisation group (RSRG) method presented by Christou and Stinchcombe (1986) in order to investigate two related models: diffusion-limited aggregation (DLA) (Witten and Sander 1981) and random lattice animals (Family 1983). It has been shown that DLA, a kinetic growth process, and lattice animals, which are equilibrium random clusters, are in distinct *static* universality classes in both the isotropic (Gould *et al* 1983) and directed (Green 1984) cases. In the following we extend the parameter space used by these authors to show that these respective systems also lie in distinct dynamic universality classes.

We consider the model of generalised DLA in two dimensions described by Gould *et al* (1983) in which the role of the seed site in the growth process of DLA is replaced by a general random cluster. It has been shown using Monte Carlo methods (Sander and Witten 1982) that the critical properties of the aggregate are unchanged in the

† In the context of this letter, the term 'dynamic' is being used in the manner discussed by Stanley *et al* (1985).

limit of large cluster growth relative to seed size. Gould *et al* (1983) used a two-parameter $b=2$ single cell RSRG method to show that these models had different static fixed points in a two-dimensional parameter space implying, for example, distinct fractal dimensions d_f (Mandelbrot 1982). A procedure analogous to this was developed by Green (1984); however she constrained the cluster to grow only eastwards or northwards favouring growth along the (1, 1) axis. Green also observed distinct static fixed points using a $b=2$ renormalisation.

Now, the dynamic exponent for these systems is the *random walk dimensionality* d_w (Christou and Stinchcombe 1986 and references therein) for random walks on the fractal space. In order to determine the structure of the parameter space from which one extracts d_w, three parameters, or fugacities, are used. A fugacity K is associated with each occupied site of the fractal cluster, another fugacity W is associated with each step of the incoming particles and a third fugacity V is associated with each step of a 'myopic ant' (de Gennes 1976) confined to walk on the cluster.

Within the RSRG procedure the square lattice is mapped onto one isomorphic to it together with an accompanied length scale dilatation of factor b. The parameters K, W, V are mapped onto K', W', V' on the new lattice. This may be represented as a flow in parameter space as pointed out above. A cluster approximation (see Stanley *et al* 1982 and references therein) is used to determine the explicit RG transformation. We seek qualitative information relating to the RG flow in parameter space, and not accurate numerical values of the dynamic exponent d_w, and we accordingly confine our attention to the simplest, $b=2$, renormalisation. To obtain reliable values for the exponents at the various fixed points one must use larger cells in order to diminish cell interfacing problems (Stanley *et al* 1982). We work on the premise that these interfacing problems do not affect the qualitative structure of the flow through parameter space, and in particular that the universality classes remain unaffected.

We evaluate the RG transformations in a manner described by Christou and Stinchcombe (1986) except that one must now consider all possible lattice animal configurations (both spanning and non-spanning) as seed sites in the $b=2$ cell. For the static parameters K, W, Gould *et al* (1983) obtain the following recursion relations:

$$K'=3K^3+K^4+6K^3W(1+W+2W^2)+4K^4W(1+2W+2W^2+4W^3) \tag{1}$$

$$W'=W^2+2W^3+5W^4+14W^5+2KW^2(1+W+3W^2+5W^3)+K^2W^2(1+2W^2). \tag{2}$$

Equation (2) contains all possible spanning random walks in a cell with some sites (non-spanning) already occupied.

Similarly for V we obtain the following transformation:

$$\begin{aligned}\tfrac{1}{2}K'V' = {} & (\tfrac{1}{3}V^2+\tfrac{1}{9}V^3+\tfrac{7}{27}V^4+\tfrac{13}{81}V^5)(\alpha K^3+2K^3W+K^3)\\ & +(\tfrac{1}{9}V^3+\tfrac{13}{81}V^5)(\alpha K^3+2K^3W+K^3)+(\tfrac{1}{4}V^2+\tfrac{1}{4}V^4)(\alpha K^3+2K^3W+K^3)\\ & +(\tfrac{1}{6}V^2+\tfrac{1}{12}V^3+\tfrac{5}{36}V^4+\tfrac{1}{9}V^5)(\beta K^4+8K^4W^2+4K^4W+K^4)\end{aligned} \tag{3}$$

where $\alpha=2W^2(1+2W)$ and $\beta=8W^3(1+2W)$.

Equations (1), (2) and (3) have the flow pattern in parameter space shown in figure 1. This demonstrates that DLA and lattice animals are indeed in distinct dynamic universality classes. The triply unstable critical fixed point E is the dynamic fixed point for DLA where one evaluates the eigenvalue $\lambda_w=(\partial V'/\partial V)_{K^*,W^*,V^*}$ from which one determines $d_w=\log\lambda_w/\log b$. The significance of the other fixed points is indicated in table 1. We note that the behaviour of the flow lines in the KW plane is identical to that found by Gould *et al* since $K'=f(K, W)$ and $W'=g(K, W)$.

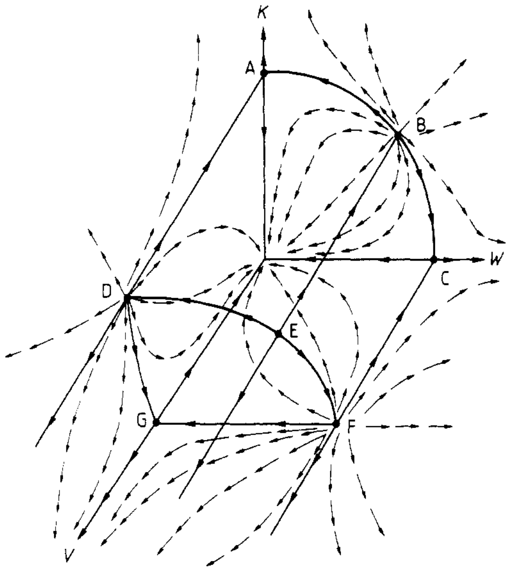

Figure 1. Renormalisation group flow diagram for transformations (1), (2), (3) and (4), (5), (6). The arrows indicate the local flow direction. The significance of the fixed points A, B, C, D, E, F, G is indicated in table 1.

Table 1. Fixed points for *isotropic* (upper of two numbers in each pair) generalised DLA and the corresponding *directed* problem (lower number in each pair) (refer to figure 1).

	Nature of fixed point	K^*	W^*	V^*
A	Lattice animals, static	0.532 0.618	0 0	0 0
B	DLA, static	0.392 0.420	0.274 0.337	0 0
C	Random walk, static	0 0	0.347 $\frac{1}{2}$	0 0
D	Lattice animals, dynamic	0.532 0.618	0 0	1.033 1.058
E	DLA, dynamic	0.392 0.420	0.274 0.337	1.048 1.080
F	Random walk, dynamic	0 0	0.347 $\frac{1}{2}$	0.963 0.950
G	Random walk, static (kinetic interpretation)	0 0	0 0	0.963 0.950

We now perform the analogous calculation for the problem of directed generalised DLA/lattice animals. The recursion relations K and W for a $b=2$ transformation are given by Green (1984):

$$K' = 2K^3 + K^4 + 4K^3W^2(1+2W) + 8K^4W^3(1+2W) + 4K^3W + 4K^4W(1+2W) \quad (4)$$

$$W' = W^2 + 2W^3 + W^2K^2 + 2KW^2(1+W) \quad (5)$$

and for the generalised directed problem we obtain the following transformation for V':

$$\begin{aligned}\tfrac{1}{2}K'V' = &(\tfrac{1}{3}V^2+\tfrac{1}{9}V^3+\tfrac{7}{27}V^4+\tfrac{13}{81}V^5)(\alpha K^3+2K^3W+K^3)\\ &+(\tfrac{1}{9}V^3+\tfrac{13}{81}V^5)(\alpha K^3+2K^3W+K^3)+(\tfrac{1}{4}V^2+\tfrac{1}{4}V^4)(0)\\ &+(\tfrac{1}{6}V^2+\tfrac{1}{12}V^3+\tfrac{5}{36}V^4+\tfrac{1}{9}V^5)(\beta K^4+8K^4W^2+4K^4W+K^4)\end{aligned} \quad (6)$$

α and β being defined as above.

These equations yield a flow diagram qualitatively equivalent to that obtained in the isotropic problem. We therefore conclude that directed DLA and directed lattice animals are also in distinct dynamic universality classes.

As a final remark we note that the fixed points F and G of figure 1 are artefacts the method in both the isotropic and directed calculations. This is so since when $K=0$ there is no cluster available on which anomalous diffusion (characterised by the parameter V) can occur. However on examining the behaviour of the recursion relations as $K\to 0$ one observes that only quasi-one-dimensional spanning clusters contribute weight to the RHS of equations (3) and (6). One may therefore identify F as the fixed point for random walks on a 1D substrate. However random walks in one dimension have identical critical behaviour to those in two dimensions ($d_w=2$), and static and dynamic random walk fixed points are equivalent ($d_f=d_w$). We therefore interpret F as the dynamic fixed point for random walks in two dimensions. Similarly, G is to be identified as the static fixed point for random walks on a one-dimensional substrate where the kinetic growth interpretation for the walks (Nakanishi and Family 1984) has been used in generating the recursion relations. The above view is consistent with the distribution of the fixed points for the other two systems.

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